MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2019/2020

ECT1026 – FIELD THEORY

(RE, TE)

24 OCTOBER 2019
9.00 a.m – 11.00 a.m
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of 8 pages including cover page with 4 questions only.

2. Attempt ALL 4 questions. All questions carry equal marks and the distribution of the marks for each question is given.

3. Please write all your answers in the answer booklet provided.

QUESTION 1

a) Convert the coordinate of the point (1,1,3) from cartesian to cylindrical.

[2 marks]

b) Transform the vector $\vec{E} = \hat{R}cos\emptyset + \hat{\theta}sin\emptyset + \hat{\emptyset}sin^2\theta$ from spherical to cylindrical and evaluate at point P = (3, π/2, π)

[5 marks]

c) Vector $\vec{D} = \hat{r}r^3$ exist in the region between two cylinder surfaces defined by r=1 and r=2, with both cylinders extend between z=0 and z=5. Evaluate $\int_v \nabla \cdot \vec{D} dV$

[6 marks]

d) For the vector field $\vec{E} = \hat{x}xy - \hat{y}(x^2+2y^2)$ find $\oint_c \vec{E} \cdot d\vec{l}$ around the triangular contour shown in Figure Q1(d) below:

[12 marks]

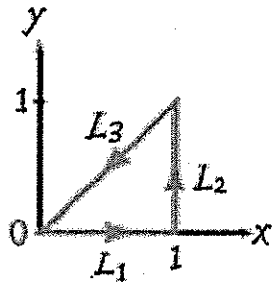

Figure Q1(d)

Continued...

QUESTION 2

a) State the two Maxwell equations in electrostatic

[2 marks]

b) Determine the total charge, Q contained in a cylindrical tube of charge oriented along the x-axis. The line charge density is $\rho_l = 3x$, where x is the distance in meters from the bottom end of the tube. The tube length is 10cm

[7 marks]

c) Find the electric field intensity, $\vec{E}$ at P (-0.2, 0, -2.3) due to a point charge of +6 nC at Q (0.2, 0.1, -2.5) in air. Assume all dimensions are in meters.

[7 marks]

d) The electric flux density inside a dielectric sphere of radius a centered at the origin is given by $\vec{D} = \hat{R}\rho_o R$ (C/m^2) where ρ_o is a constant. Find the total charge, Q inside the sphere.

[9 marks]

Continued...

QUESTION 3

a) Given vectors $\vec{A} = \hat{r}(cos\emptyset + 3z) - \hat{\emptyset}(2r + 4sin\emptyset) + \hat{z}(r - 2z)$ and $\vec{B} = -\hat{r}sin\emptyset + \hat{z}cos\emptyset$, find θ_{AB} at $(2, \frac{\pi}{2}, 0)$

[5 marks]

b) Transform vector $\vec{B} = \hat{y}(x^2+y^2+z^2) - \hat{z}(x^2+y^2)$ into spherical coordinates and evaluate at point P = (-1,0,2).

[5 marks]

c) Find the gradient of the scalar function $V=xy^2z^4$

[3 marks]

d) A coil of 300 turns is wound uniformly over an air ring having a mean circumference of 400 mm and a uniform cross sectional area of 500 mm^2. If the current through the coil is 4 A, calculate:

i) The magnetic field strength, H

[3 marks]

ii) The magnetic flux density, B

[3 marks]

iii) The total magnetic flux, Φ

[3 marks]

e) Explain why ferromagnetic material is always preferred as compared to paramagnetic and diamagnetic materials in magnetic systems.

[3 marks]

Continued...

QUESTION 4

a) Given vectors $\vec{A} = \hat{x}2 - \hat{y}3 + \hat{z}$, $\vec{B} = \hat{x}2 - \hat{y} + \hat{z}3$ and $\vec{C} = \hat{x}2 + \hat{y} - \hat{z}$. Show that $\hat{C}$ is perpendicular to both $\hat{A}$ and $\hat{B}$.

[4 marks]

b) Convert the point P = $(2, \frac{\pi}{4}, -4)$ from Cylindrical to Cartesian coordinate.

[2 marks]

c) Find the total charge, Q contained in a round in a round top cone defined by R$\leq 2\ m$ and $0 \leq \theta \leq \pi/4$, given that $\rho_v = 30R^2cos^2\theta$ (C/m^3)

[6 marks]

d) An iron core toroid of relative permeability 3000 has mean radius R=80 mm and a circular cross section with radius b = 25 mm as shown in figure below. An air gap of length 3 mm exists and a current flows in the winding to produce a magnetic flux of 10^{-5} Wb. Neglecting leakage and using mean path length, find:

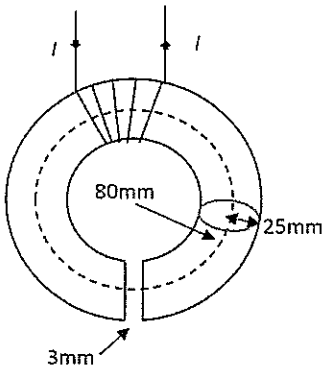

Figure Q4(b)

i) The reluctance of the air gap and of the iron core.

[4 marks]

ii) Magnetic flux density, B and magnetic field intensity, H of the air gap.

[4 marks]

e) Sketch the hysteresis loop of a ferromagnetic materials and label the following points:
i) Saturation
ii) Retentivity
iii) Coercivity

[5 marks]

Continued...

Appendix A : Physical constants, Vectors and Coordinate Transformations

Elementary charge	e	1.60×10^{-19} C
Permittivity constant	ε_0	8.85×10^{-12} F/m
Permeability constant	μ_0	1.26×10^{-6} H/m

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Differential length	$\hat{x}dx + \hat{y}dy + \hat{z}dz$	$\hat{r}dr + \hat{\phi}rd\phi + \hat{z}dz$	$\hat{R}dR + \hat{\theta}Rd\theta + \hat{\phi}R\sin\theta d\phi$
Differential surface areas	$d\vec{s}_x = \hat{x}dydz$ $d\vec{s}_y = \hat{y}dxdz$ $d\vec{s}_z = \hat{z}dxdy$	$d\vec{s}_r = \hat{r}rd\phi dz$ $d\vec{s}_\phi = \hat{\phi}drdz$ $d\vec{s}_z = \hat{z}rdrd\phi$	$d\vec{s}_R = \hat{R}R^2\sin\theta d\theta d\phi$ $d\vec{s}_\theta = \hat{\theta}R\sin\theta dRd\phi$ $d\vec{s}_\phi = \hat{\phi}RdRd\theta$
Differential volume	$dxdydz$	$rdrd\phi dz$	$R^2\sin\theta dRd\theta d\phi$

Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to Cylindrical	$r = \sqrt{x^2 + y^2}$ $\phi = \tan^{-1}(y/x)$ $z = z$	$\hat{r} = \hat{x}\cos\phi + \hat{y}\sin\phi$ $\hat{\phi} = -\hat{x}\sin\phi + \hat{y}\cos\phi$ $\hat{z} = \hat{z}$	$A_r = A_x\cos\phi + A_y\sin\phi$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$ $A_z = A_z$
Cylindrical to Cartesian	$x = r\cos\phi$ $y = r\sin\phi$ $z = z$	$\hat{x} = \hat{r}\cos\phi - \hat{\phi}\sin\phi$ $\hat{y} = \hat{r}\sin\phi + \hat{\phi}\cos\phi$ $\hat{z} = \hat{z}$	$A_x = A_r\cos\phi - A_\phi\sin\phi$ $A_y = A_r\sin\phi + A_\phi\cos\phi$ $A_z = A_z$
Cartesian to Spherical	$R = \sqrt{x^2 + y^2 + z^2}$ $\theta = \tan^{-1}(\sqrt{x^2 + y^2}/z)$ $\phi = \tan^{-1}(y/x)$	$\hat{R} = \hat{x}\sin\theta\cos\phi + \hat{y}\sin\theta\sin\phi + \hat{z}\cos\theta$ $\hat{\theta} = \hat{x}\cos\theta\cos\phi + \hat{y}\cos\theta\sin\phi - \hat{z}\sin\theta$ $\hat{\phi} = -\hat{x}\sin\phi + \hat{y}\cos\phi$	$A_R = A_x\sin\theta\cos\phi + A_y\sin\theta\sin\phi + A_z\cos\theta$ $A_\theta = A_x\cos\theta\cos\phi + A_y\cos\theta\sin\phi - A_z\sin\theta$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$
Spherical to Cartesian	$x = R\sin\theta\cos\phi$ $y = R\sin\theta\sin\phi$ $z = R\cos\theta$	$\hat{x} = \hat{R}\sin\theta\cos\phi + \hat{\theta}\cos\theta\cos\phi - \hat{\phi}\sin\phi$ $\hat{y} = \hat{R}\sin\theta\sin\phi + \hat{\theta}\cos\theta\sin\phi + \hat{\phi}\cos\phi$ $\hat{z} = \hat{R}\cos\theta - \hat{\theta}\sin\theta$	$A_x = A_R\sin\theta\cos\phi + A_\theta\cos\theta\cos\phi - A_\phi\sin\phi$ $A_y = A_R\sin\theta\sin\phi + A_\theta\cos\theta\sin\phi + A_\phi\cos\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$
Cylindrical to Spherical	$R = \sqrt{r^2 + z^2}$ $\theta = \tan^{-1}(r/z)$ $\phi = \phi$	$\hat{R} = \hat{r}\sin\theta + \hat{z}\cos\theta$ $\hat{\theta} = \hat{r}\cos\theta - \hat{z}\sin\theta$ $\hat{\phi} = \hat{\phi}$	$A_R = A_r\sin\theta + A_z\cos\theta$ $A_\theta = A_r\cos\theta - A_z\sin\theta$ $A_\phi = A_\phi$
Spherical to Cylindrical	$r = R\sin\theta$ $\phi = \phi$ $z = R\cos\theta$	$\hat{r} = \hat{R}\sin\theta + \hat{\theta}\cos\theta$ $\hat{\phi} = \hat{\phi}$ $\hat{z} = \hat{R}\cos\theta - \hat{\theta}\sin\theta$	$A_r = A_R\sin\theta + A_\theta\cos\theta$ $A_\phi = A_\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$

Appendix B: Gradient, Divergence, Curl and Laplacian Operators

Cartesian coordinate (x, y, z)

$$\nabla V = \hat{x}\frac{\partial V}{\partial x} + \hat{y}\frac{\partial V}{\partial y} + \hat{z}\frac{\partial V}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \hat{x}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{y}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{z}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

Cylindrical coordinate (r, ϕ, z)

$$\nabla V = \hat{r}\frac{\partial V}{\partial r} + \hat{\phi}\frac{1}{r}\frac{\partial V}{\partial \phi} + \hat{z}\frac{\partial V}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \frac{1}{r}\begin{vmatrix} \hat{r} & \hat{\phi}r & \hat{z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix} = \hat{r}\left(\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right) + \hat{\phi}\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \hat{z}\frac{1}{r}\left[\frac{\partial}{\partial r}\left(rA_\phi\right) - \frac{\partial A_r}{\partial \phi}\right]$$

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

Spherical coordinate (R, θ, ϕ)

$$\nabla V = \hat{R}\frac{\partial V}{\partial R} + \hat{\theta}\frac{1}{R}\frac{\partial V}{\partial \theta} + \hat{\phi}\frac{1}{R\sin\theta}\frac{\partial V}{\partial \phi}$$

$$\nabla \cdot \vec{A} = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2 A_R) + \frac{1}{R\sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{R\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \vec{A} = \frac{1}{R^2\sin\theta}\begin{vmatrix} \hat{R} & \hat{\theta}R & \hat{\phi}R\sin\theta \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_R & RA_\theta & (R\sin\theta)A_\phi \end{vmatrix}$$

$$= \hat{R}\frac{1}{R\sin\theta}\left[\frac{\partial}{\partial \theta}\left(A_\phi \sin\theta\right) - \frac{\partial A_\theta}{\partial \phi}\right] + \hat{\theta}\frac{1}{R}\left[\frac{1}{\sin\theta}\frac{\partial A_R}{\partial \phi} - \frac{\partial}{\partial R}\left(RA_\phi\right)\right] + \hat{\phi}\frac{1}{R}\left[\frac{\partial}{\partial R}\left(RA_\theta\right) - \frac{\partial A_R}{\partial \theta}\right]$$

$$\nabla^2 V = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{R^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$

Appendix C : Table of Integrals

$$\int \sin^2\theta d\theta = \frac{\theta}{2} - \frac{1}{4}\sin 2\theta + \text{constant}$$

$$\int \cos^2\theta d\theta = \frac{\theta}{2} + \frac{1}{4}\sin 2\theta + \text{constant}$$

$$\int \sin\theta\cos^2\theta d\theta = -\frac{1}{3}\cos^3\theta + \text{constant}$$

$$\int \cos\theta\sin^4\theta d\theta = \frac{1}{5}\sin^5\theta + \text{constant}$$

$$\int \sin 2\theta d\theta = -\frac{1}{2}\cos 2\theta + \text{constant}$$

$$\int x\sqrt{a^2 - x^2}\, dx = -\frac{1}{3}\left(a^2 - x^2\right)^{3/2} + \text{constant}$$

$$\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\tan^{-1}\left(\frac{x}{\sqrt{a^2 - x^2}}\right) + \text{constant}$$

$$\int \frac{x}{\sqrt{a^2 - x^2}} dx = -\sqrt{a^2 - x^2} + \text{constant}$$

$$\int \frac{dz}{\left(r^2 + z^2\right)^{3/2}} = \frac{z}{r^2\sqrt{r^2 + z^2}} + \text{constant}$$

$$\int \frac{rdr}{\left(r^2 + z^2\right)^{3/2}} = \frac{-1}{\sqrt{r^2 + z^2}} + \text{constant}$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + \text{constant}$$

End of Paper